

Investigation of Prediction Methods for Tip Rake Propellers

Rhena Klose¹, Reinhard Schulze¹, Katrin Hellwig-Rieck¹

¹Schiffbau-Versuchsanstalt Potsdam (SVA), Potsdam, Germany

ABSTRACT

Tip Rake Propellers are characterized by a distinct rake to the pressure side at the blade tip. The tip vortex cavitation shall be reduced by this rake. Therefore the propeller can afford a high load at the tip without having an increased intensity of tip vortex cavitation. Regarding their operating principle it is supposed that Tip Rake Propellers are more sensitive to changes in inflow and circulation than conventional propellers. The knowledge about the Reynolds number effect is therefore necessary to ensure a fair comparison with different propeller designs.

In this paper the application of already existing Reynolds number correction procedures ((ITTC 2014), (Lerbs 1951, Schmidt 1972) and the Strip Method of (Streckwall, Greitsch, Scharf 2013)) on experimentally obtained propeller characteristics of conventional propellers and Tip Rake Propellers are described. On basis of this analysis an improved Reynolds number correction method was developed at SVA that is presented in this paper, too. It is demonstrated that the SVA Method fulfills the criteria of a good Reynolds number correction procedure. Also the open water characteristics of Tip Rake Propellers can be scaled to full-scale Reynolds number.

Furthermore, the influence of the scale on the cavitation behavior in simulated model and full-scale wake fields is presented. The cavitation phenomena show big differences even for small wake field differences. A change of inflow has a different effect on the pressure fluctuations for each propeller. The necessity of the correct simulation of the wake field for the evaluation of the cavitation behavior is shown, especially for Tip Rake Propellers which operating principle is based on influencing the pressure distribution at the tip.

Keywords

Tip Rake Propellers, open water characteristics, Reynolds number correction methods, scale effects, cavitation behavior.

1 INTRODUCTION

The propeller design plays a significant role for the energetic and acoustic (vibrations and noise) optimization of ships. It is common that for new-build or redesign projects propellers are designed in competition by different parties and compared in open water tests as well as propulsion and cavitation tests. In this context different opinions are often expressed regarding the scale effects

(Reynolds number effect) especially when different propeller geometries are compared. This includes for example propellers with modified tips like Tip Rake Propellers with a distinct rake to the pressure side at the blade tip or Tip Fin Propellers (Kappel Propellers) with the rake to the suction side (see figure 1). The present studies are confined to propellers with tip rakes to the pressure side (Tip Rake Propellers).

Tip Rake Propellers shall reduce the tip vortex cavitation by dispersing vortices from the tip. This type of propeller can therefore afford a higher load at the tip with the same or better cavitation properties and high efficiency in comparison to a conventional propeller (Dang 2004). This feature is increasingly integrated in modern propeller designs and is a challenge for model basins to guarantee reliable prognoses for this propeller type.

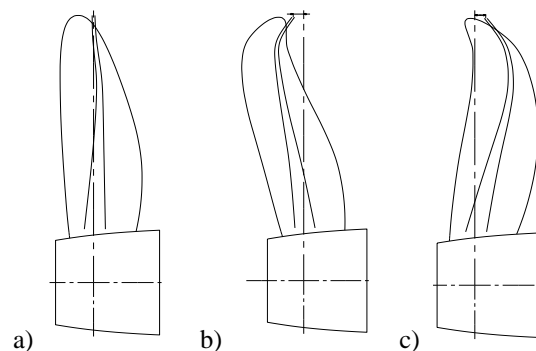


Figure 1 a) rake = 0 (conventional propeller), b) rake to suction side (Tip Fin / Kappel Propeller), c) rake to pressure side (Tip Rake Propeller), (Dang 2004)

The operating principle of a Tip Rake Propeller leads to the assumption that this type of propeller is more sensitive to changes in inflow and circulation than conventional propellers. The knowledge about the Reynolds number effect is therefore necessary to ensure a fair comparison of different propeller designs.

In the present paper a selection of already existing Reynolds number correction procedures are applied on experimentally obtained propeller characteristics of conventional propellers and Tip Rake Propellers and analyzed by their repeatability and reliability. One of the investigated Tip Rake Propellers (P1727) was provided to ITTC as the benchmarking test case “Unconventional Propeller: PPTC II” (SVA 2016). On basis of this analysis

an improved Reynolds number correction method by SVA is presented.

It is investigated if the methods are independent of the realized Reynolds number in the open water tests by comparing the scaled propeller characteristics obtained for three different Reynolds number ranges for each propeller. Furthermore, the accuracy of the correction methods is checked by means of full-scale CFD calculations.

In a further step the influence of the scale on the cavitation behavior (tested by cavitation observations and propeller induced pressure fluctuations) in simulated model and full-scale wake fields is presented.

2 Existing Reynolds number correction procedures

The open water test and its full-scale correction are of high importance since it forms the basis for all further propulsion prognoses.

Due to the inability of realizing Reynolds number similarity in open water tests, the propeller characteristics have to be corrected to full-scale. Over the years, different Reynolds number correction procedures were developed. The requirements for a good correction procedure are a quick and easy implementation as well as repetitive and reliable results. In detail, this means that the results have to be independent from the Reynolds number achieved in open water test and from the propeller geometry (different types of propellers must not be advantaged or disadvantaged) (Helma 2015).

Existing Reynolds number correction procedures are amongst others the 1978 ITTC Performance Prediction Method (ITTC 2014), the Method of Lerbs/Schmidt (Lerbs 1951, Schmidt 1972) and the Strip Method of HSVA, TUHH und MMG (Streckwall, Greitsch, Scharf 2013).

2.1 1978 ITTC Performance Prediction Method

This method is still the standard procedure for the correction of scale effects on propeller characteristics. However, this method is often accused of not meeting today's requirements, especially for modern propeller designs. It was developed at a time when due to a lack of computational assistance simple calculation methods were required. This is also the reason why the propeller geometry is taken into account only at a reference radius of $r/R = 0.75$. This might disadvantage propellers with unconventional blade geometries like propellers with modified tips.

2.2 Method of Lerbs/Schmidt

This method relies on the method of the "equivalent airfoil section" by applying the knowledge from airfoils to the propeller blade. The open water coefficients K_T and K_Q are converted to the airfoil coefficients C_L and C_D in dependency of the angle of attack α .

For the calculation Lerbs (1951) assumed that for small roughness the scale effect only affects C_D . However, bigger roughness interferes with the circulation around the airfoil and due to the reduced induced velocity the lift coefficient C_L increases while the angle of attack α

decreases. This dependency of C_L , C_D as well as α on the friction coefficient C_F was investigated by Schmidt (1972) with the help of extensive model tests and implemented in the method of Lerbs. Similar to the 1978 ITTC Method this method refers only to the propeller radius $r/R = 0.75$.

2.3 Strip Method of HSVA, TUHH, MMG

This strip method was developed by HSVA, TUHH and MMG for the European research project PREFUL (Streckwall, Greitsch, Scharf 2013). It is based on a new friction characteristic for propellers obtained by extensive RANS calculations. The friction correction is done for radial sections (strips) taking into account the chord length, the pitch and the rake of the whole propeller blade. This approach promises a good assessment of effects that arise from a certain propeller geometry. However, it has to be noted that the method is still in a development stage and that the scaling of unconventional propellers with respect to blade area and rake has not yet been investigated.

3 NEW METHOD BY SVA

The new Reynolds number correction procedure developed by SVA (Schulze 2017) follows mainly the basic principles of the 1978 ITTC Performance Prediction Method, especially the consideration of only global propeller parameters at a reference radius of $r/R = 0.75$. Four modifications were applied which relate primarily to the calculation of the friction resistance of the foil sections.

3.1 Form factor for foil sections

The drag coefficient of the foil section proposed by the 1978 ITTC Method is expressed as the friction coefficient of the smooth plate C_F and a form factor:

$$C_D = 2 \cdot \left(1 + 2 \cdot \frac{t}{c} \right) \cdot C_F \quad (1)$$

A more precise version of the form factor for foil sections is given by Torenbeek (1982):

$$C_D = 2 \cdot \left(1 + 2.7 \cdot \frac{t}{c} + 100 \cdot \left(\frac{t}{c} \right)^4 \right) \cdot C_F \quad (2)$$

This formula is used for the new method.

3.2 Friction coefficient for transition zone

The friction coefficient of the smooth plate is given by the 1978 ITTC Method for model scale with:

$$C_{FM} = \frac{0.044}{(Re)^{\frac{1}{6}}} - \frac{5}{(Re)^{\frac{2}{3}}} \quad (3)$$

The resulting curve is presented in figure 2.

Reynolds numbers of model tests are often located in the transition zone of laminar to turbulent flow. In this region formula 3 might not represent the friction characteristic for real conditions correctly (see figure 2). In this respect, the requirement of the 1978 ITTC Method of $Re_{c,0.75} > 2 \cdot 10^5$ for open water tests is too low.

For a better consideration of the friction conditions on a model propeller especially in the transition zone the following friction curve is suggested:

$$C_{FM} = 0.3 \cdot Re^{-1/3} \quad Re \leq 10^6 \quad (4)$$

$$C_{FM} = 0.003 \quad 10^6 \leq Re \leq 1.7 \cdot 10^6 \quad (5)$$

$$C_{FM} = \frac{3.913}{(\ln(Re))^{2.58}} - \frac{1700}{Re} \quad 1.7 \cdot 10^6 \leq Re \quad (6)$$

The related curve is shown in figure 2 as thick black line.

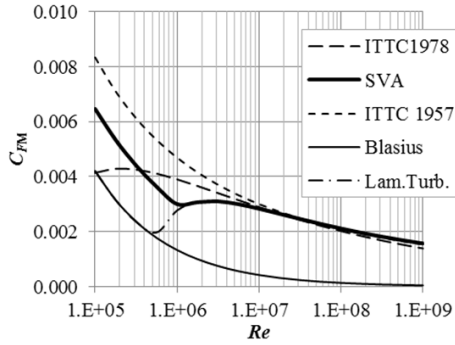


Figure 2 Friction characteristics for the smooth plate in the transition zone of laminar to turbulent flow

3.3 Influence of roughness on full-scale

According to the 1978 ITTC Method the Reynolds number Re is not necessary for the calculation of the full-scale friction coefficient of the smooth plate:

$$C_{FS} = \left(1.89 + 1.62 \cdot \log \frac{c}{k_P} \right)^{-2.5} \quad (7)$$

This assumption is considered insufficient. A better consideration of the Reynolds number and surface properties like roughness provides Schlichting, Gersten (2000):

$$\sqrt{\frac{2}{C_{FS}}} = \frac{1}{\kappa} \cdot \log \left(\frac{C_{FS}}{2} Re \right) + 5.0 - \frac{1}{\kappa} \cdot \log(3.4 + k_{tech}^+) \quad (8)$$

Where $k_{tech}^+ = 0.001 Re \cdot k_P / c$ and $\kappa = 0.41$.

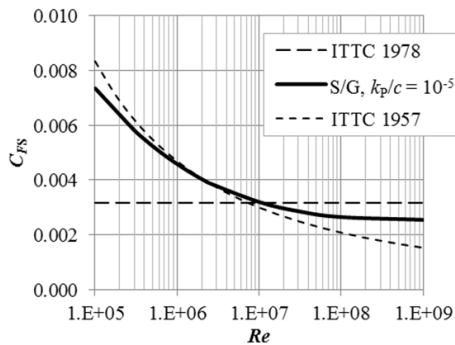


Figure 3 Friction characteristics for the smooth plate with consideration of surface roughness $k_P/c = 10^{-5}$

In figure 3 the Schlichting/Gersten curve (S/G) is presented for $k_P/c = 10^{-5}$ in comparison to the 1978 ITTC Method according to formula (8) and the ITTC 1957 curve.

3.4 Variable boundary layer at dynamometer

For the standard open water test in the towing tank the dynamometer is arranged behind the propeller in order to have homogenous inflow (see figure 4). Nevertheless, it is suspected that there is a retaining effect due to the Reynolds number depending boundary layer at the gondola of the dynamometer. This retaining effect can be included in the evaluation procedure by introducing a correction factor for the advance coefficient J :

$$J = \frac{(1 - w_{ow}) \cdot V}{n \cdot D} \quad (9)$$

$$w_{ow} = \frac{0.08 \left[\frac{m^2}{s^2} \right]}{(n_M \cdot D_M)^2} \quad (10)$$

Where w_{ow} is higher for small Reynolds numbers and therefore bigger boundary layers occur. The suggested correction is depending on the test-setup and dynamometer that is used for open water tests. It must be investigated carefully.

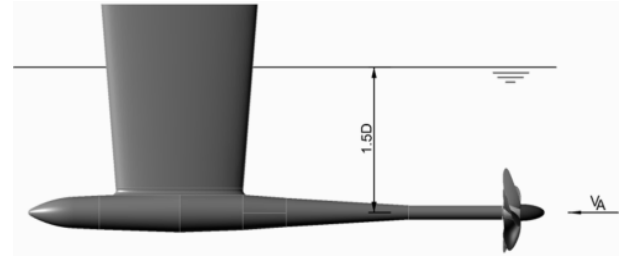


Figure 4 Test set-up for open water tests with the dynamometer behind the propeller

The further proceeding is according to the 1978 ITTC Method. Investigations regarding the benefit of applying the corrections for defined radial sections (strip method) led to the result that no additional improvement was achieved so that it was omitted for reasons of simplicity.

4 APPLICATION TO CONVENTIONAL AND TIP RAKE PROPELLERS

4.1 Subjects of study

Based upon the original propeller design data of a single screw and a twin screw vessel different Tip Rake Propellers have been designed and manufactured for testing. The design specification was only limited to the diameter and the blade number of the original (conventional) propellers. Hence conclusions about the effectiveness of the tip rake cannot be drawn but from Okazaki (2015) it is known that for moderate tip rake distributions K_T and K_Q decrease with increasing rake angles while η_0 is nearly constant.

The main data of the conventional propellers (CP) and Tip Rake Propellers (TRP) are given in table 1. The propellers P1664, P1727 and P1730 belong to the single screw vessel and the propellers P1720 and P1729 to the twin screw vessel.

Table 1 Main data of the propellers

		P1664	P1727	P1730	P1720	P1729
		CP	TRP	TRP	CP	TRP
D	[mm]	238.64	238.64	238.64	180.00	180.00
$P_{0.75}/D$	[-]	0.8356	0.8014	0.8749	0.9160	0.9123
A_E/A_0	[-]	0.4183	0.4438	0.3787	0.7054	0.6663
$c_{0.75}$	[mm]	52.648	55.631	47.523	52.884	52.494
θ_{EXT}	[°]	24.030	25.650	21.472	34.614	29.649
Z	[-]	4	4	4	5	5

In comparison to the original propellers, especially the TRPs P1727 and P1729 are characterized by an extensive rake at the tip while the propeller P1730 has a moderate tip rake (figure 5).

Side views of the propellers are shown in figure 6.

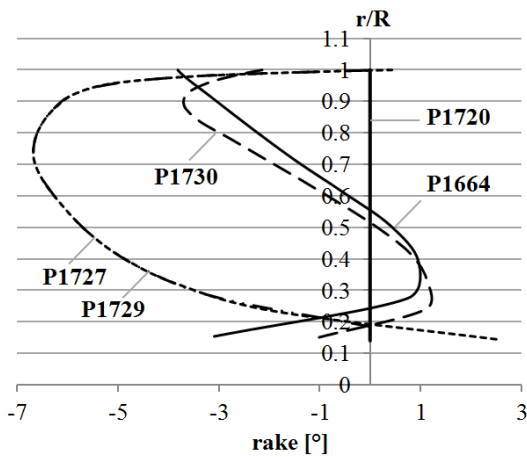


Figure 5 Rake characteristics of the propellers

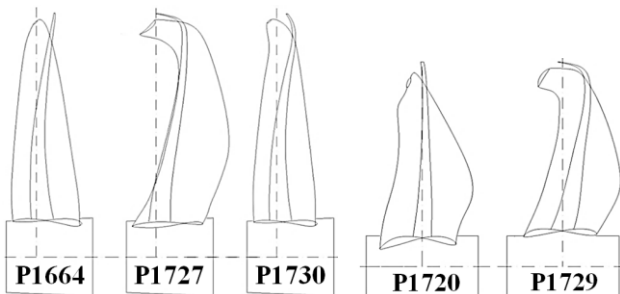


Figure 6 Side views of the propellers

4.2 Open water tests

The open water characteristics of the introduced propellers were determined for three typical model scale

Reynolds number ranges each. The measured characteristics of thrust K_T and torque K_Q show a dependency on Reynolds number $Re_{c0.75}$ for all propellers so that also the resulting open water efficiencies η_O vary for same advance coefficients J . An example is presented for the TRP P1727 (PPTC II) in figure 7. J_{OP} marks the advance coefficient for the operation point of the ship which is primarily the interesting point for further investigations.

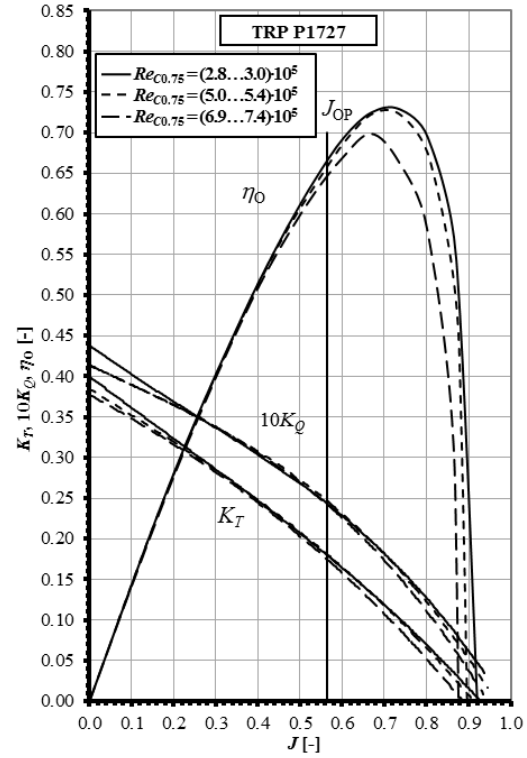


Figure 7 Open water characteristics of TRP P1727 for three Reynolds number ranges

The differences in open water efficiency at J_{OP} for the smallest and biggest Reynolds number range are given for all propellers in table 2.

Table 2 Differences of open water efficiency between smallest and biggest tested Reynolds number range at the ship's operation point

	P1664	P1727	P1730	P1720	P1729
	CP	TRP	TRP	CP	TRP
$\Delta\eta_O(J_{OP})$ [%]	1.7	1.5	2.1	0.6	1.5

For the propellers of the single screw vessel P1664, P1727 and P1730 as well as the P1729 of the twin screw vessel the difference is around 1.5 – 2 % likewise for the CP and the TRPs, while the difference for the CP of the twin screw vessel P1720 is 0.6 %. It has to be mentioned that the propellers of the single screw vessel have comparatively small blade area ratios and chord lengths which makes them more affected by scale effects in model tests.

4.3 Full-scale prognosis

For each of the five propellers the Reynolds number correction procedures presented in the chapters 2 and 3 are applied on the open water characteristics obtained for three typical Reynolds number ranges. The results for the TRP P1727 and the SVA Method are exemplified in figure 8. At the advance coefficient J_{OP} the resulting open water efficiencies η_o are now on the same level.

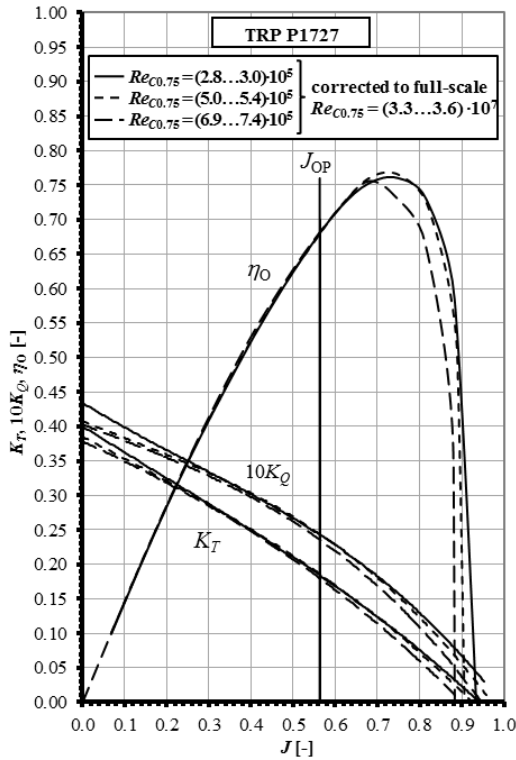


Figure 8 Open water characteristics of TRP P1727 obtained for three different Reynolds number ranges scaled to full-scale by SVA Method

In table 3 the differences in open water efficiency at J_{OP} for the smallest and biggest Reynolds number range after scaling to full-scale Reynolds number range are given for all propellers and Reynolds number correction procedures.

Table 3 Differences of open water efficiency between smallest and biggest tested Reynolds number range at the ship's operation point after scaling to full-scale

	P1664	P1727	P1730	P1720	P1729
	CP	TRP	TRP	CP	TRP
1978 ITTC					
$\Delta\eta_o(J_{OP})$ [%]	1.5	1.1	1.8	0.4	1.3
Lerbs/Schmidt					
$\Delta\eta_o(J_{OP})$ [%]	1.0	0.5	0.4	0.2	0.8
Strip Method					
$\Delta\eta_o(J_{OP})$ [%]	2.0	2.4	2.6	1.0	2.1
SVA Method					
$\Delta\eta_o(J_{OP})$ [%]	0.6	0.1	0.2	0.9	0.5

Differences below 1 % are achieved for all types of propellers with the Lerbs/Schmidt Method and the SVA Method. In this case, the quality criteria of independency from the Reynolds number and from the propeller geometry are fulfilled. In contrary, differences over 1 % as for the ITTC Method and the Strip Method are not satisfying.

4.4 Viscous open water calculations

The absolute correctness of the methods is not possible to validate since no full-scale measurements are available. A first hint can be given by computational fluid dynamic calculations (CFD) for full-scale.

First, the model scale open water characteristics were calculated by means of CFD for the five propellers. The differences between EFD (experimental fluid dynamics) and CFD are given for two advance coefficients around the operation point in table 4. The differences vary between 0.3 – 2.7 %.

Table 4 Differences of open water efficiency between EFD and CFD (model scale) for $J = 0.5$ and 0.7

	P1664	P1727	P1730	P1720	P1729
	CP	TRP	TRP	CP	TRP
$\Delta\eta_o(J=0.5)$ [%]	0.3	1.3	1.7	0.7	0.3
$\Delta\eta_o(J=0.7)$ [%]	0.4	2.0	2.3	2.7	1.3

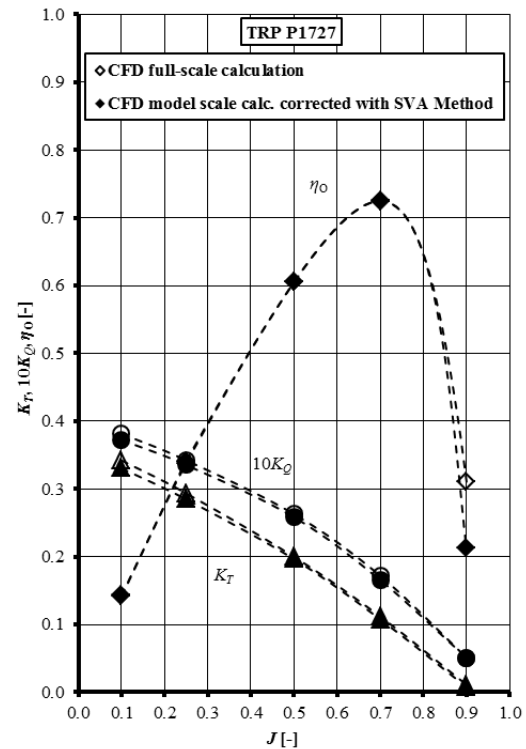


Figure 9 Open water characteristics of TRP P1727 obtained for model scale CFD calculations Reynolds number corrected by SVA Method and full-scale CFD-calculations

The same points were calculated for full-scale and compared to the Reynolds number corrected model scale CFD calculations. An example is given in figure 9 for the TRP P1727 and the SVA Method.

In table 5 the differences in open water efficiency at two advance coefficients are given for all propellers and Reynolds number correction procedures.

In contrast to the results in chapter 4.3 the biggest differences up to 2.3 % result from the Lerbs/Schmidt Method and the smallest differences below 1.2 % from the Strip Method which admittedly itself is based on a friction characteristic obtained by CFD calculations. Similarly good results as for the Strip Method are provided for all investigated types of propellers by applying the SVA Method. Even if this check is no substitute for a full-scale validation it can give a first hint about the correctness of a method.

The SVA Method fulfills thereby the requirements for a good correction procedure of giving correct results (validated with CFD calculations for full-scale) independently from the Reynolds number achieved in open water test for CPs as well as TRPs.

Table 5 Differences of open water efficiency between Reynolds number corrected CFD model scale calculation and CFD full-scale calculation for $J = 0.5$ and 0.7

	P1664	P1727	P1730	P1720	P1729
	CP	TRP	TRP	CP	TRP
1978 ITTC					
$\Delta\eta_O(J=0.5)$ [%]	0.9	0.7	0.7	0.9	0.1
$\Delta\eta_O(J=0.7)$ [%]	1.8	1.3	1.1	1.0	0.1
Lerbs/Schmidt					
$\Delta\eta_O(J=0.5)$ [%]	1.0	0.6	1.1	1.4	0.6
$\Delta\eta_O(J=0.7)$ [%]	2.3	1.6	2.1	2.2	1.3
Strip Method					
$\Delta\eta_O(J=0.5)$ [%]	0.2	0.3	0.1	0.3	0.5
$\Delta\eta_O(J=0.7)$ [%]	0.7	0.7	0.1	0.0	1.2
SVA Method					
$\Delta\eta_O(J=0.5)$ [%]	0.6	0.1	0.2	0.9	0.4
$\Delta\eta_O(J=0.7)$ [%]	1.2	0.0	0.0	1.0	1.0

5 PROGNOSIS OF CAVITATION PROPERTIES

Besides the propulsion prognosis also the prognosis of the cavitation properties plays an important role in the decision-making process for propeller designs. This includes the prediction of the type and extent of cavitation phenomena and the induced pressure fluctuations for full-scale. The current methods and procedures are analyzed regarding their validity for TRPs. Especially the effect of changings in the inflow of the propeller between model

scale and full-scale wake field on the cavitation phenomena is investigated.

Therefore cavitation tests were carried out in the cavitation tunnel with the dummy models of a single screw and a twin screw vessel. The propellers P1664, P1727 and P1730 belong to the single screw vessel and the propellers P1720 and P1729 to the twin screw vessel. The test set-up in the cavitation tunnel for the single screw vessel is presented in figure 10.

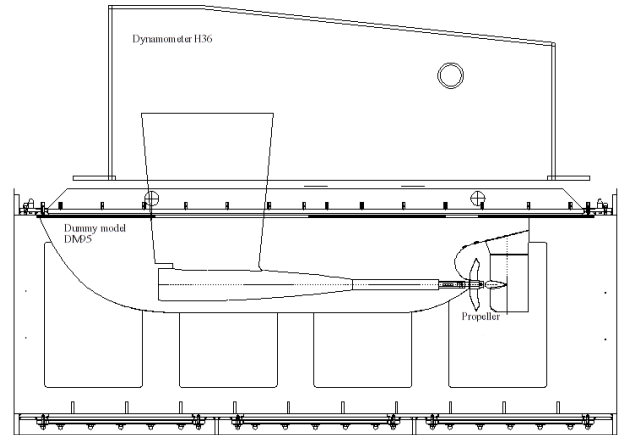


Figure 10 Test set-up in the test section for the single screw vessel with dummy model, dynamometer, propeller and rudder

The propellers were tested in three dimensional wake fields, generated by the dummy model and additional wire screens for model and full-scale. The calculated wake fields for the single screw vessel are shown in figure 11. Due to the different boundary layer characteristic the inflow in model scale is more affected, especially for the inner radii, than for full-scale conditions. For the twin screw vessel this effect is less distinctive but not negligible.

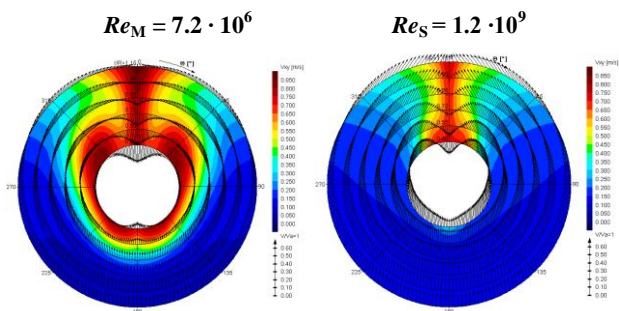


Figure 11 Wake fields for the single screw vessel in model scale (left) and full-scale (right)

5.1 Cavitation phenomena

Looking at the cavitation behavior of the propellers the cavitation extents at the operation points are bigger for the model scale wake field (see figure 12).

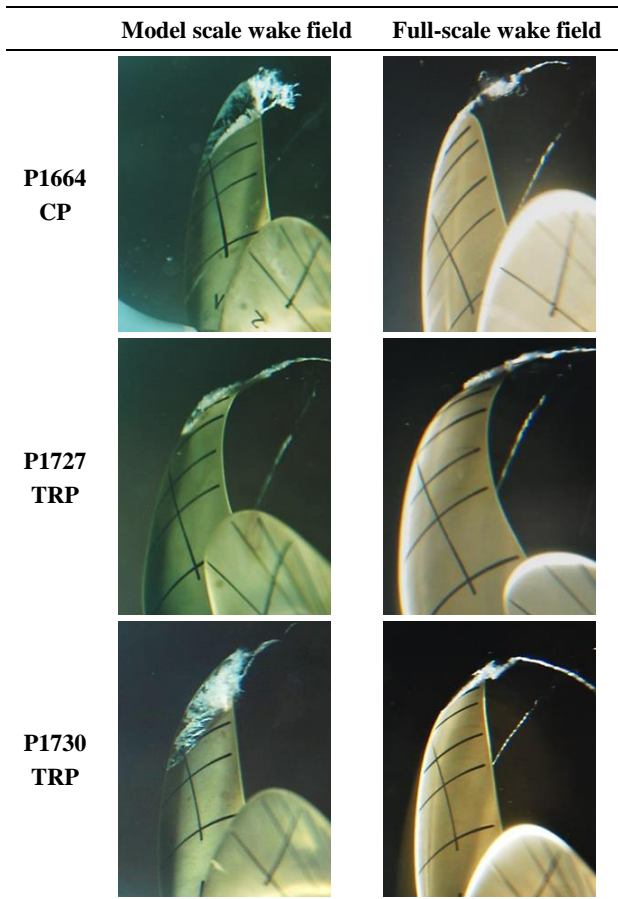


Figure 12 Cavitation observations at operation point for model and full-scale wake field

This shows that for the evaluation of the cavitation properties of a propeller design the simulation of the correct wake field in cavitation tests is important. For TRPs this is more important since their operating principle is based on a reduction of the cavitation extent at the blade tip.

5.2 Induced pressure fluctuations

The same observations are made for the induced pressure fluctuations. In figure 13 the amplitudes are given as full-scale values for 10 pressure sensor positions at the dummy model above the propeller (see figure 14). The amplitudes have been measured in the simulated model scale as well as full-scale wake field and scaled to the full-scale operation point afterwards.

The full-scale pressure amplitudes are higher for the propellers operating in the model scale wake field than for the full-scale wake field. Interestingly, the TRP P1727 with the distinctive tip rake is less affected as the propellers P1664 and P1730. For example at position 6 (propeller plane, 12 o'clock) the pressure amplitudes are around 2.5 times higher in the 1st harmonic and around 5 times higher in the 2nd harmonic for the propellers P1664 and P1730 between model scale and full-scale wake field. Instead, for the propeller P1727 it is only slightly higher.

This shows that the propellers are differently affected by the changes in inflow irrespective if it is a CP or a TRP. It is therefore important to test all propellers in the correct wake field.

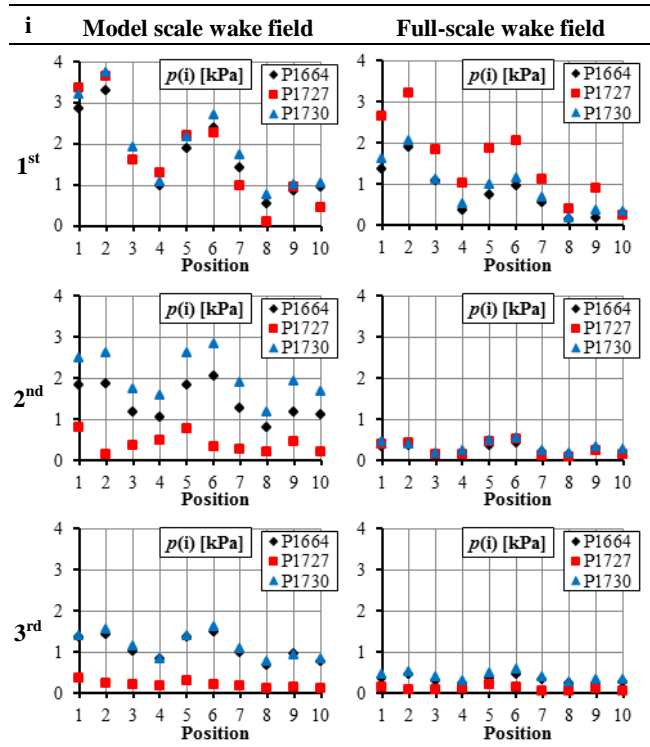


Figure 13 Amplitudes of full-scale pressure fluctuations at operation point for model and full-scale wake field

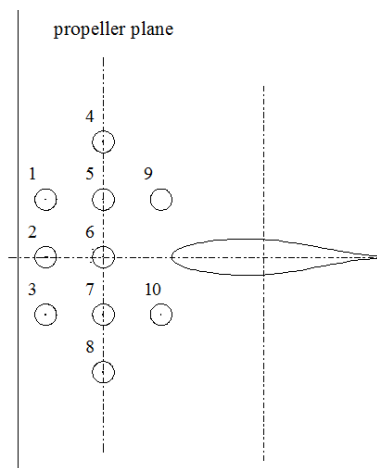


Figure 14 Positions of pressure sensors 1 to 10 in the dummy model of the single screw vessel (top view)

6 CONCLUSIONS

In the paper it was shown that the investigated existing Reynolds number correction procedures do not meet all requirements of a good correction method which was shown for conventional propellers as well as Tip Rake Propellers. A new method, based on the 1978 ITTC Method was developed. It is called SVA Method and provides satisfying results for all investigated propeller

designs. The quality criteria of independency from the model scale Reynolds number and from the propeller geometry (conventional propellers and Tip Rake propellers) were fulfilled. The adaptability to other unconventional propeller geometries needs to be proved.

Regarding the cavitation tests the investigations showed that small changes in inflow due to scale effects lead to different results for the cavitation observations and pressure fluctuation measurements. It is therefore important to simulate the correct wake field, especially for competing propeller designs since the propellers can be affected differently by the inflow.

With the obtained knowledge and improvements the same test procedures and scaling methods are valid for conventional propellers and Tip Rake Propellers.

ACKNOWLEDGEMENT

The authors express their gratitude to the Federal Ministry for Economic Affairs and Energy for sponsoring the research project Tip Rake, to the shipyard Lürssen-Kröger and the design office Neptun Ship Design for providing their ship designs and to Mecklenburger Metallguss (MMG) for designing the Tip Rake Propeller P1730.

NOMENCLATURE

CFD	computational fluid dynamics
CP	conventional propeller
EFD	experimental fluid dynamics
TRP	Tip Rake Propeller
A_E/A_0	[-] expanded blade area ratio
C_F	[-] friction coefficient
C_D	[-] drag coefficient
C_L	[-] lift coefficient
c	[m] chord length
D	[m] propeller diameter
i	[-] order of harmonics
J	[-] advance coefficient $V_A / (D n)$
n	[1/s] number of revolutions
Q	[Nm] torque
K_Q	[-] torque coefficient $Q / (\rho n^2 D^5)$
K_T	[-] thrust coefficient $T / (\rho n^2 D^4)$
k_p	[m] blade roughness 10E-6 m
k_{tech}^+	[-] technical roughness
P	[m] propeller pitch
p	[Pa] pressure
r	[m] local radius
R	[m] propeller radius
Re	[-] Reynolds number VL / ν
T	[N] propeller thrust

t	[m]	blade thickness
V_A	[m/s]	inflow velocity
w_{ow}	[-]	correction number for open water tests
Z	[-]	blade number
α	[°]	angle of attack
η_O	[-]	open water efficiency
θ_{EXT}	[°]	skew angle
κ	[-]	Karman constant
ρ	[kg/m ³]	density of fluid

REFERENCES

- ITTC (2014). '1978 ITTC Performance Prediction Method'. Recommended Procedures and Guidelines 7.5-02-03-01.4.
- Dang, J. (2004). 'Improving Cavitation Performance with new Blade Sections for Marine Propellers'. 9th International Symposium on Practical Design of Ships and other Floating Structures, Lübeck-Travemünde.
- Helma, S. (2015). 'An Extrapolation Method Suitable for Scaling Propellers of any Design'. 4th International Symposium on Marine Propulsors, Austin.
- Lerbs, H. W. (1951). 'On the Effects of Scale and Roughness on Free Running Propellers'. J.A.S.N.E., Report 63.
- Okazaki, A., Yamasaki S., Kawanami, Y., Ukon, Y., Ando, J. (2015). 'The Effect of Tip Rake on Propeller Open Water Efficiency and Propulsive Efficiency'. 4th International Symposium on Marine Propulsors, Austin.
- Schlichting, H.; Gersten, K. (2000). 'Boundary-Layer Theory'. Springer, 8th edition.
- Schmidt, D. (1972). 'Einfluss der Reynoldszahl und der Rauigkeit auf die Propellercharakteristik, berechnet nach der Methode des äquivalenten Profils'. Schiffbauforschung 11.
- Schulze, R. (2017). 'A new friction correction method for the open water characteristics of propellers'. STG Annual Book, Hamburg.
- Streckwall, H., Greitsch, L., Scharf, M. (2013). 'An advanced Scaling Procedure for Marine Propellers'. 3rd International Symposium on Marine Propulsors, Launceston.
- SVA (2016). 'Unconventional Propeller: PPTC II'. [http://www.sva-potsdam.de/itc-benchmark/.](http://www.sva-potsdam.de/itc-benchmark/)
- Torenbeek, E. (1982). 'Synthesis of Subsonic Airplane Design'. Delft University Press, Delft.