# SVA High-Speed Propeller Series 

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#### Abstract

Propellers are still the most used propulsion system for fast ships. These propellers are mostly combined with inclined shafts. The variation in the transversal velocity of the propeller in oblique inflow causes considerable changes in the profile angle of attack, which leads to cavitation especially if the propeller rotates downward. A propeller series has been designed and investigated, to get data for the pre-design of high-speed propellers and for the propulsion prognosis in the early design state for fast ships.


## 1. INTRODUCTION

Characteristics of modern propellers are necessary for the propulsion prognosis and the design of propellers for fast ships ( $\mathrm{V}_{\mathrm{S}}=30$ to 50 kts ). The propellers are working mostly in oblique flow due to the shaft inclination and the trim of the ship.

Propellers for fast ships with a high efficiency and good cavitation behaviour had been developed and optimised in cooperation with AIR. Lifting surface methods (VORTEX) and viscous flow methods (COMET, CFX5) have been used for the propeller design. Four propellers with different design strategies have been manufactured and investigated in open water and cavitation tests. In addition the influence of a cup at the trailing edge at the propeller characteristic and the cavitation behaviour had been tested.

The optimised propeller was the starting point for the SVA High-Speed Propeller series. Twelve model propellers have been manufactured for the propeller series. The open water tests have been carried out in the towing tank and in the cavitation tunnel. The test results have been analysed with a multidimensional Tschebyscheff approximation. A program has been developed to calculate the propeller characteristic for given propeller data and to choose the propeller for given working parameters.

## 2. PROPELLER PRE-DESIGN

Different propellers have been designed and tested to define a basic propeller for the SVA High Speed Propeller Series. In a first step the use of a cupped propeller has been investigated.

### 2.1 Cupped propeller

The propeller cup is a widely used technique to correct and enhance vessel performance. The cup is a curvature of the trailing edge to the pressure side (Denny, S. (1989)) (Figure 1). The definition of cup (e.g. light, heavy) varies greatly between the companies and also between projects. A specification of the cup has been done in accordance to the maximum thickness of the profile (Denny, S. (1989)) (Table 1).


Fig. 1. Profile with a cup (definition of the cup according to U.S. Navy)

Table 1. Definition of the cup [1]

| cup | curvature of trailing <br> edge $x[m m]$ | radius of the cup <br> $R_{\mathrm{C}}[\mathrm{mm}]$ |
| :--- | :---: | :---: |
| light | $0.33\left(\mathrm{t}_{\max }\right)_{0.7 \mathrm{R}}$ | $2.5\left(\mathrm{t}_{\max }\right)_{0.7 \mathrm{R}}$ |
| moderate | $0.67\left(\mathrm{t}_{\max }\right)_{0.7 \mathrm{R}}$ | $5.0\left(\mathrm{t}_{\max }\right)_{0.7 \mathrm{R}}$ |
| heavy | $1.00\left(\mathrm{t}_{\max }\right)_{0.7 \mathrm{R}}$ | $7.5\left(\mathrm{t}_{\max }\right)_{0.7 \mathrm{R}}$ |

The propeller P1359 has been designed and tested to study the influence of the cup on the open water characteristics and cavitation behaviour. A moderate and a heavy cup have been investigated (Figure 2).


Fig. 2. Model propeller P1359 with two cup variants
The cup effects an increasing of the propeller thrust and torque coefficients (Figures 3 and 4). The efficiency of the propeller P1359-2 with the heavy cup is smaller than the efficiency of the propeller P1359 without cup or with moderate cup (P1359-1).


Fig. 3. Influence of the cup at the propeller characteristics, without cavitation


Fig. 4. Influence of the cup at the propeller characteristics, $\sigma_{V}=0.94$

The cup effects a releasing of the entrance range of the suction side. The suction side cavitation is smaller at a propeller with cup (Figure 5). On the other side the tests with different cavitation numbers showed that small cavitation phenomenon at the suction side of the cup results in an early thrust reduction of the propeller (Figure 4 and Table 2). The Figure 6 presents the calculated pressure distributions at the blades of the P1359-2 with heavy cup for the advance coefficients $\mathbf{J}=0.546$ and 1.13. The maximum low pressure appears in the range of the cup at the suction side of the blade. On the pressure side of the blade appears a large over pressure. That means, that the thrust of a propeller with heavy cup will generate mainly in the aft part of the blade. That's why small cavitation at the cup leads to a thrust breakdown of the propeller.


Fig. 5. Model propeller P1359 without cup and with two cup variants, cavitation behaviour at $\varphi=0^{\circ}, \mathrm{C}_{\mathrm{TP}}=1.15, \sigma_{\mathrm{V}}=0.94$

Table 2. Inception of the thrust breakdown due to cavitation

| Propeller |  | $\varphi=0^{\circ}$ | $\varphi=8^{\circ}$ | $\varphi=13^{\circ}$ |
| :--- | :--- | :---: | :---: | :---: |
| P1359 | $\mathrm{C}_{\mathrm{TP}}=1.15$ | $\sigma_{\mathrm{V}}=1.37$ | $\sigma_{\mathrm{V}}=1.37$ | $\sigma_{\mathrm{V}}=1.37$ |
| P1359-1 | $\mathrm{C}_{\mathrm{TP}}=1.15$ | $\sigma_{\mathrm{V}}=1.84$ | $\sigma_{\mathrm{V}}=1.84$ | $\sigma_{\mathrm{V}}=1.84$ |
| P1359-2 | $\mathrm{C}_{\mathrm{TP}}=1.15$ | $\sigma_{\mathrm{V}}=2.03$ | $\sigma_{\mathrm{V}}=1.97$ | $\sigma_{\mathrm{V}}=1.94$ |



Fig. 6. Calculated pressure distributions for the propeller P1359-2 with heavy cup, shaft inclination $0^{\circ}$

### 2.2 Propeller geometry variation

As a result of the tests with cupped propellers it was decided to design the propeller series without a cup. Different propellers have been designed by Potsdam Model Basin and AIR. The main focus of the propeller optimisation was the cavitation behaviour (minimising of significant cavitation and associated thrust breakdown, minimising of erosion risk).

The Figure 7 shows for example the variation of the design parameters chord length, pitch and camber for the propeller variants P1381 and P1382. The cavitation tests with these two propellers showed better cavitation behaviour of the propeller P1382. The cavitation had to be improved only in the root region of this propeller.


Fig. 7. Design parameters of the propellers P1381 and P1382


Fig. 8. Cavitation behaviour, propellers P 1381 and $\mathrm{P} 1382, \varphi=13^{\circ}, \sigma_{\mathrm{v}}=0.948$

## 3. SVA HIGH-SPEED PROPELLER SERIES

The 3-bladed SVA High-Speed Propeller series consists of 12 models (4 propellers for each expanded area ratio of $\mathrm{A}_{\mathrm{E}} / \mathrm{A}_{0}=0.9,1.1$ and 1.3) with the diameter of $\mathrm{D}=220 \mathrm{~mm}$, hub diameter ratio $\mathrm{d}_{\mathrm{h}} / \mathrm{D}=0.1818$ and pitch ratios $\mathrm{P}_{0.7} / \mathrm{D}=1.0,1.2,1.4$ and 1.6 (Table 3).

The Figure 9 shows the calculated pressure distribution of the propeller P1390 ( $\mathrm{P}_{\mathrm{m}} / \mathrm{D}=$ $1.353, \mathrm{~A}_{\mathrm{E}} / \mathrm{A}_{0}=1.10$ ) in the design point. The cavitation behaviour and the risk of thrust breakdown have been investigated for different propellers of the series. At first sheet cavitation appears at the leading edge of the suction side. Tip vortex cavitation starts at lower cavitation numbers. A thrust breakdown was measured for the propellers with an area ration $\mathrm{A}_{\mathrm{E}} / \mathrm{A}_{0}=1.10$, if sheet cavitation appears on $2 / 3$ of the suction side area.

Table 3. SVA High-Speed Propeller series

|  | $\mathrm{A}_{\mathrm{E}} / \mathrm{A}_{0}=0.90$ | $\mathrm{~A}_{\mathrm{E}} / \mathrm{A}_{0}=1.10$ | $\mathrm{~A}_{\mathrm{E}} / \mathrm{A}_{0}=1.30$ |
| :---: | :---: | :---: | :---: |
| $\mathrm{P}_{0.7} / \mathrm{D}=1.00$ | P 1383 | P 1384 | P 1385 |
| $\mathrm{P}_{\mathrm{m}} / \mathrm{D}=0.963$ | P 1386 | P 1387 | P 1388 |
| $\mathrm{P}_{0.7} / \mathrm{D}=1.20$ |  |  |  |
| $\mathrm{P}_{\mathrm{m}} / \mathrm{D}=1.158$ | P 1389 | P 1390 | P 1391 |
| $\mathrm{P}_{0.7} / \mathrm{D}=1.40$ |  |  |  |
| $\mathrm{P}_{\mathrm{m}} / \mathrm{D}=1.353$ | P 1392 | P 1393 | P 1394 |
| $\mathrm{P}_{0.7} / \mathrm{D}=1.60$ | $\mathrm{P}_{\mathrm{m}} / \mathrm{D}=1.544$ |  |  |



Fig. 9. Pressure distribution at the propeller $\mathrm{P} 1390\left(\mathrm{P}_{0.7} / \mathrm{D}=1.4, \mathrm{~J}=1.085, \varphi=0^{\circ}\right)$, calculated with CFX 5
The Figure 10 shows as an example the measured open water characteristics of the model propeller P1390 at different cavitation numbers and the Figure 11 presents the cavitation inception curves for this propeller.


Fig. 10. Open water characteristics, propeller P1390 $\left(\mathrm{P}_{\mathrm{m}} / \mathrm{D}=1.353, \mathrm{~A}_{\mathrm{E}} / \mathrm{A}_{0}=1.10\right), \varphi=0^{\circ}$


Fig. 11. Cavitation inception curves, propeller P1390 $\left(\mathrm{P}_{\mathrm{m}} / \mathrm{D}=1.353, \mathrm{~A}_{\mathrm{E}} / \mathrm{A}_{0}=1.10\right), \varphi=0^{\circ}$

### 3.1 Open water and cavitation tests

The open water tests have been carried out in the SVA's towing tank ( $280 \times 9 \times 4.5 \mathrm{~m}$ ) and in the test section No. $2(850 \mathrm{~mm} \times 850 \mathrm{~mm}$ ) of the cavitation tunnel K15A.

Tests have been carried out at atmospheric pressure and at the cavitation numbers $\sigma_{\mathrm{v}}=4.5$, $1.5,1.0$ und 0.75 with a shaft inclination of 0,6 and 12 degrees. The range of the Reynolds number based on 0.7 R chord length was $\mathrm{Re}=(1.9-6.1)^{*} 10^{6}$. The measured propeller thrust and torque had been corrected by the measured system friction and dummy hub torque and thrust.
The open water test results were faired and the thrust coefficients $\mathrm{K}_{\mathrm{TP}}$, torque coefficients $\mathrm{K}_{\mathrm{Q}}$ and open water efficiency $\eta_{\mathrm{O}}$ have been calculated as function of the advance coefficient J , the cavitation number $\sigma_{\mathrm{v}}$ and the shaft inclination $\varphi$. The Figures 12 to 23 present the open water characteristics of the propellers P1384, P1387, P1390 and P1393 with an area ratio $\mathrm{A}_{\mathrm{E}} / \mathrm{A}_{\mathrm{O}}=1.10$ at the cavitation numbers $\sigma_{\mathrm{V}}=4.5,1.5,1.0$ und 0.75 and a shaft inclination of 0,6 and 12 degrees.


Fig. 12. HSP 3.110, $\varphi=0^{\circ}, \sigma_{v}=4.5$


Fig. 14. HSP 3.110, $\varphi=12^{\circ}, \sigma_{v}=4.5$


Fig. 16. HSP 3.110, $\varphi=6^{\circ}, \sigma_{v}=1.5$


Fig. 18. HSP 3.110, $\varphi=0^{\circ}, \sigma_{v}=1.0$


Fig. 13. HSP $3.110, \varphi=6^{\circ}, \sigma_{v}=4.5$


Fig. 15. HSP 3.110, $\varphi=0^{\circ}, \sigma_{v}=1.5$


Fig. 17. $\operatorname{HSP} 3.110, \varphi=12^{\circ}, \sigma_{V}=1.5$


Fig.19. HSP 3.110, $\varphi=6^{\circ}, \sigma_{V}=1.0$


Fig. 20. HSP 3.110, $\varphi=12^{\circ}, \sigma_{v}=1.0$


Fig. 22. $\operatorname{HSP} 3.110, \varphi=6^{\circ}, \sigma_{v}=0.75$


Fig. 21. $\operatorname{HSP} 3.110, \varphi=0^{\circ}, \sigma_{V}=0.75$


Fig. 23. HSP 3.110, $\varphi=12^{\circ}, \sigma_{v}=0.75$

The open water tests with the propeller series show the following tendencies regarding the influence of the oblique inflow and the area ratio on the propeller characteristics.

Oblique inflow: The propeller thrust and torque coefficients are increasing at low thrust loading coefficients in an oblique inflow. The cavitation is increasing at oblique inflow in the range where the blades work again the oblique inflow. The thrust breakdown is starting at larger cavitation numbers with increasing of the shaft inclination.

Area ratio: The increasing of the area ratio effects an increasing of the torque coefficient. The increasing of the torque coefficient is larger at high thrust loading coefficients than at low thrust loading coefficients. The thrust coefficient is growing at high thrust loading coefficients with an increasing if the area ratio. The efficiency is decreasing with increasing of the area ratio. The thrust breakdown due to cavitation starts at larger cavitation numbers if the area ratio will be increased. The Figures 24 to 27 show the influence of the area ratio on the open water characteristics of the propellers with a pitch ratio $\mathrm{P}_{\mathrm{m}} / \mathrm{D}=1.353$ and 1.544 without and with cavitation.


Fig. 24. Influence of the area ratio, propellers with $\mathrm{P}_{\mathrm{m}} / \mathrm{D}=1.353$, shaft inclination $0^{\circ}$, without cavitation


Fig. 25. Influence of the area ratio, propellers with $\mathrm{P}_{\mathrm{m}} / \mathrm{D}=1.353$, shaft inclination $0^{\circ}, \sigma_{\mathrm{v}}=0.75$


Fig. 26. Influence of the area ratio, propellers with $\mathrm{P}_{\mathrm{m}} / \mathrm{D}=1.544$, shaft inclination $0^{\circ}$, without cavitation


Fig. 27. Influence of the area ratio, propellers with $\mathrm{P}_{\mathrm{m}} / \mathrm{D}=1.544$, shaft inclination $0^{\circ}, \sigma_{\mathrm{v}}=0.75$

### 3.2 Calculation of propeller characteristics using multidimensional Chebyshev approximation

The method of the so called "polynomial coefficients" is a well known method to describe open water characteristics of propellers by some main propeller parameters as number of blades, pitch ratio and area ratio for specialised propeller families like "Wageningen BB propeller series". Based on measurements for selected propellers of a propeller family the method of "polynomial coefficients" can be interpreted from the mathematical point of view as an interpolation, extrapolation or approximation problem respectively. The application of approximation techniques was preferred to overcome the expected numerical problems by the application of interpolation strategies for the High-Speed Propeller series. This guaranteed the highest smoothness of resulting functions and was applied first time for the approximation of open water characteristics of ducted propellers in the Potsdam Model Basin (Schulze, R. (2000), (1996), (1999)). Based on the mathematical theory of Hilbert spaces the following results are summarised.

Denote $\mathrm{I}_{\mathrm{i}}:=\left[\mathrm{a}_{\mathrm{i}}, \mathrm{b}_{\mathrm{i}}\right] \subseteq \mathrm{R}, \mathrm{i}=1, \ldots, \mathrm{n}$ one dimensional interval of real numbers and $\mathrm{I}:=\mathrm{I}_{1} \times \mathrm{I}_{2} \times \ldots \times \mathrm{I}_{\mathrm{n}} \subseteq \mathrm{R}^{\mathrm{n}}$ a n -dimensional interval and $\mathrm{H}_{\mathrm{i}}$ the Hilbert spaces of all square integral functions over $I_{i}$ as well as $H$ the Hilbert space of all over I square integral functions with the scalar products $(x, y)=\int_{\mathrm{Ii}} \mathrm{x}(\mathrm{t}) \cdot \mathrm{y}(\mathrm{t}) \cdot \mathrm{g}_{\mathrm{i}}(\mathrm{t}) \cdot \mathrm{dt}$ with a weighting function $\mathrm{g}_{\mathrm{i}}: \mathrm{I}_{\mathrm{i}} \rightarrow \mathrm{R}$.

The scalar product $\left({ }^{*},{ }^{*}\right)$ in H is then defined by a multi-integral. The following functions $\left\{\Phi_{\mathrm{j}}^{\mathrm{i}}\right\}, \mathrm{j}=0, \ldots$ are to be orthogonal basis in the Hilbert spaces $\mathrm{H}_{\mathrm{i}}$. The functions

$$
\begin{equation*}
\left\{\Phi_{\mathrm{j} 1, \mathrm{j} 2, \ldots, \mathrm{jn}}=\Phi_{\mathrm{j} 1}^{1} \cdot \Phi_{\mathrm{j} 2}^{2} \cdot \ldots \cdot \Phi_{\mathrm{jn}}^{\mathrm{n}}\right\} \tag{1}
\end{equation*}
$$

constitute a orthogonal basis in the space H with the multi index $(\mathrm{j} 1 \mathrm{j}, \mathrm{j} 2, \ldots, \mathrm{jn})$ in which $\mathrm{j}_{\mathrm{i}} \in$ ( $0, \ldots . .$. ),
$\mathrm{i}=1, \ldots, \mathrm{n}$ realised by a convenient counting. Based on these fundamentals each function $\mathrm{x} \in \mathrm{H}$ is representable as an abstract Fourier series

$$
\begin{equation*}
\mathrm{x}=\sum_{(\mathrm{j} 1, \mathrm{j} 2, \ldots, \mathrm{jn})} \mathrm{a}_{\mathrm{j} 1, \mathrm{j}, \ldots, \ldots \mathrm{j}} \cdot \Phi_{\mathrm{j} 1, \mathrm{j}, \ldots, \ldots \mathrm{jn}} \tag{2}
\end{equation*}
$$

with the abstract Fourier coefficients $\mathrm{a}_{\mathrm{j} 1, \mathrm{j} 2, \ldots, \mathrm{jn}} \in \mathrm{R}$. Based on these assumptions it follows for the Fourier coefficients

$$
\begin{equation*}
\mathrm{a}_{\mathrm{j} 1, \mathrm{j}, \ldots, \ldots \mathrm{jn}}=\left(\mathrm{x}, \Phi_{\mathrm{jl} 1, \mathrm{j} 2, \ldots, \mathrm{jn}}\right) /\left(\Phi_{\mathrm{j} 1, \mathrm{j} 2, \ldots, \mathrm{jn}}, \Phi_{\mathrm{j} 1, \mathrm{j} 2, \ldots, \mathrm{jn}_{n}}\right)^{1 / 2} . \tag{3}
\end{equation*}
$$

For orthonormal systems of functions $\left\{\Phi_{\mathrm{j} 1, \mathrm{j} 2, \ldots, \mathrm{jn}}=\Phi_{\mathrm{j} 1}^{1} \cdot \Phi_{\mathrm{j} 2}^{2} \cdot \ldots \cdot \Phi_{\mathrm{jn}}^{\mathrm{n}}\right\} \quad$ i.e. $\left(\Phi_{\mathrm{j} 1, \mathrm{j} 2, \ldots, \mathrm{jn}}, \Phi_{\mathrm{j} 1, \mathrm{j} 2, \ldots, \mathrm{jn}}\right)^{1 / 2}=1 \quad$ and $\quad\left(\Phi_{\mathrm{j} 1, \mathrm{j} 2, \ldots, \mathrm{jn}}, \Phi_{\mathrm{k} 1, \mathrm{k} 2, \ldots, \mathrm{kn}}\right)^{1 / 2}=0$ for different multi indices ( $\mathrm{j} 1, \mathrm{j} 2, \ldots, \mathrm{jn}$ ) and ( $\mathrm{k} 1, \mathrm{k} 2, \ldots, \mathrm{kn}$ ) it follows the Parseval's equation

$$
\begin{equation*}
(x, x)=\sum_{(\mathrm{j}, \mathrm{j}, 2, \ldots, \mathrm{jn})}\left(\mathrm{a}_{\mathrm{j} 1, \mathrm{j}, \ldots, \ldots, \mathrm{jn}}\right)^{2} . \tag{4}
\end{equation*}
$$

A finite sum of the series (2) is the best approximation in the sense of the norm in the Hilbert space generated by the corresponding scalar product. For an arbitrary $x \in H$ it follows now

$$
\begin{equation*}
\left(x, \phi_{11,12, \ldots, j n}\right)=\iint_{11.12} \ldots \int_{\text {in }} x\left(t_{1}, t_{2}, \ldots, t_{n}\right) \cdot \phi_{j 1}^{1}\left(t_{1}\right) \cdot \phi_{j 2}^{2}\left(t_{2}\right) \cdot \ldots \cdot \phi_{\text {in }}^{n}\left(t_{n}\right) \cdot g_{1}\left(t_{1}\right) \cdot g_{2}\left(t_{2}\right) \cdot \ldots \cdot g_{n}\left(t_{n}\right) \cdot d t_{1} \cdot d t_{2} \cdot \ldots \cdot d t_{n} \tag{5}
\end{equation*}
$$

This multi-integral must be computed in general by numerical methods. The boundaries of intervals in general are predefined by the problem. The best weighting functions are often known by experience. The following samples give as usual such functions:

Weighting function interval orthogonal functions (polynomials)

$$
\begin{array}{lll}
\mathrm{g}(\mathrm{t})=1 & {[-1,1]} & \text { Legendre-polynomials } \\
\mathrm{g}(\mathrm{t})=\left(1-\mathrm{t}^{2}\right)^{-1 / 2} & {[-1,1]} & \text { Chebyshev-polynomials of } 1^{\text {st }} \text { type } \\
\mathrm{g}(\mathrm{t})=\mathrm{e}^{-\mathrm{t}^{*} \mathrm{t}} & (-\infty, \infty) & \text { Hermite-polynomials } \\
\mathrm{g}(\mathrm{t})=\mathrm{e}^{-\mathrm{t}} \cdot \mathrm{t}^{\alpha}, \alpha>-1 & {[0, \infty)} & \text { Laguerre-polynomials }
\end{array}
$$

Exclusive for the Chebyshev-polynomials simultaneously the approximation succeeds in the sense of quadratic mean and uniformly (Graf Finck von Finckenstein (1977)). Hence in the following the orthonormal function system of Chebyshev-polynomials of $1^{\text {st }}$ type was preferred. The Chebyshev-polynomials are defined over the interval $[-1,1]$ by
$\mathrm{T}_{\mathrm{i}}(\mathrm{t})=\cos (\mathrm{i} \cdot \arccos (\mathrm{t}))$
which is equivalent to the recursion

$$
\begin{equation*}
\mathrm{T}_{0}(\mathrm{t})=1, \mathrm{~T}_{1}(\mathrm{t})=\mathrm{t}, \ldots, \mathrm{~T}_{\mathrm{n}+1}(\mathrm{t})=2 \cdot \mathrm{~T}_{\mathrm{n}}(\mathrm{t})-\mathrm{T}_{\mathrm{n}-1}(\mathrm{t}) \tag{7}
\end{equation*}
$$

For an explicit representation of the first 11 Chebyshev-polynomials see e.g. (Philippow, E. (1963) page 272). For these polynomials hence follows:

$$
\begin{align*}
& \mathrm{a}_{0}=\frac{2}{\pi} \cdot \int_{[-1,1]} \mathrm{x}(\mathrm{t}) \cdot \mathrm{g}(\mathrm{t}) \cdot \mathrm{dt}  \tag{8}\\
& \mathrm{a}_{\mathrm{i}}=\frac{1}{\pi} \cdot \int_{[-1,1]} \mathrm{x}(\mathrm{t}) \cdot \mathrm{T}_{\mathrm{i}}(\mathrm{t}) \cdot \mathrm{g}(\mathrm{t}) \cdot \mathrm{dt}, \mathrm{i}=1,2, \ldots
\end{align*}
$$

By the substitution
$\mathrm{s}=\frac{1}{2} \cdot(\mathrm{a}+\mathrm{b})+\frac{1}{2} \cdot(\mathrm{~b}-\mathrm{a}) \cdot \mathrm{t} \leftrightarrow \mathrm{t}=(2 \cdot \mathrm{~s}-(\mathrm{a}+\mathrm{b})) /(\mathrm{b}-\mathrm{a}) \quad$,
for $\mathrm{s} \in[\mathrm{a}, \mathrm{b}]$ and $\mathrm{t} \in[-1,1]$ the interval $[-1,1]$ maps to the interval $[\mathrm{a}, \mathrm{b}]$ and vice versa. Or in other words the function $\mathrm{y}:[\mathrm{a}, \mathrm{b}] \rightarrow \mathrm{R}$ represented by a function $\mathrm{x}:[-1,1] \rightarrow \mathrm{R}$ by
$x(t)=y(s)=y\left(\frac{1}{2} \cdot(a+b)+\frac{1}{2} \cdot(b-a) \cdot t\right)$
and vice versa
$y(s)=x(t)=x((2 \cdot s-(a+b)) /(b-a))$.
Hence the approximation problem based on Chebyshev-polynomials can be extended for arbitrary finite intervals.

In difference to an interpolation procedure by this approximation procedure the knowledge of boundaries for the intervals are needed, but it is possible to extrapolate the approximating functions outside these boundaries. For the realization of these approximation techniques used in general special numerical quadrature formulas.

The open water characteristics of the High-Speed Propeller series measured in the towing tank and the cavitation tunnel of the SVA with the values $\left[J, \mathrm{P}_{0.7}, \mathrm{~A}_{\mathrm{E}} / \mathrm{A}_{0}, \sigma_{V}, \varphi\right]$ as parameters give a five dimensional approximation problem for functions. The corresponding Fourier coefficients for Chebyshev polynomials were derived by the described method of the order three to seven (polynomial order differ from the considered parameter) in the multi interval $\mathrm{I}=([0,1.7] ;[1.0,1.6] ;[0.9,1.3] ;[0.75,4.5] ;[0,12.0])$.

### 3.3 Software HSPOPT

For the calculation of the open water characteristic in dependence of the five parameters $\left[J, \mathrm{P}_{0.7}, \mathrm{~A}_{\mathrm{E}} / \mathrm{A}_{0}, \sigma_{\mathrm{v}}, \varphi\right]$ based on test results and for the design process for High-Speed Propellers to determine the unknown pitch for given delivered power or thrust the calculation software HSPOPT was designed by SVA.

The following example shows the possibilities of the software HSPOPT.
For a given fast ship a propeller with a diameter $\mathrm{D}=1.33 \mathrm{~m}$, a pitch ratio $\mathrm{P}_{0.7} / \mathrm{D}=1.049$ and an area ratio of $\mathrm{A}_{\mathrm{E}} / \mathrm{A}_{0}=1.13$ was considered. For the advance coefficient $\mathrm{J}=0.80$ and the cavitation number $\sigma_{\mathrm{V}}=0.90$ a propeller thrust coefficient $\mathrm{K}_{\mathrm{TP}}=0.160$ and a torque coefficient $\mathrm{K}_{\mathrm{Q}}=0.0321$ were needed.

The software program HSPOPT based on the SVA High-Speed Propeller series calculated a propeller HSP 3.113 with a pitch of $\mathrm{P}_{0.7} / \mathrm{D}=1.0724$ (Figure 30). The drawing of this propeller is given in the Figure 31. Figure 32 shows the calculated open water characteristic.


Fig. 28. Dialog for the test example with the software


Fig. 29. Propeller drawing for the calculated propeller HSP 3.113 with $\mathrm{P}_{0.7} / \mathrm{D}=1.0724$


Fig. 30. Calculated open water characteristic HSP 3.113, $\mathrm{P}_{0.7} / \mathrm{D}=1.0724$, shaft inclination $6^{\circ}, \sigma_{\mathrm{v}}=0.90$

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